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A critical study of a relativistic wave equation of unique mass and spin-1 particles proposed by Khalil

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Abstract. It is shown that the unique mass spin-1 wave equation proposed by Khalil is not derivable from an invariant Lagrangian and no charge can be defined for it.

In a previous paper Khalil (1979) has given a relativistic wave equation of the Gel'fand-Yaglom form

$$(-\mathrm{i}\Gamma_\mu \partial^\mu + m)\psi(x) = 0 \tag{1}$$

with matrix Γ_0 given in table 15 (p 661) of the above paper. The wavefunction $\psi(x)$ transforms according to the representation

$$[\dot{\tau}_2 \sim (-1, 2)] \oplus [\tau_1 \sim (0, 2)] \oplus [\tau_2 \sim (1, 2)] \oplus [\tau_1' \sim (0, 2)] \oplus [\tau_3 \sim (1, 3)] \tag{2}$$

with interlocking scheme

$$\begin{tikzcd}
& \dot{\tau}_2 \sim (-1, 2) \arrow[dl, leftrightarrow] \arrow[dr, leftrightarrow] & \\
(0, 2) \sim \tau_1 \arrow[dr, leftrightarrow] & & \tau_1' \sim (0, 2) \arrow[dl, leftrightarrow] \\
& \tau_2 \sim (1, 2) \arrow[d, leftrightarrow] & \\
& \tau_3 \sim (1, 3) &
\end{tikzcd} \tag{3}$$

where we have labelled the irreducibles of the representation using the Gel'fand-Yaglom notation. The two notations are related as follows:

$$l_0 = n - k \qquad l_1 = k + n + 1 \tag{4}$$

where the pair (k, n) stands for the ordinary way of labelling representations and the pair (l_0, l_1) for the Gel'fand-Yaglom way.

The purpose of this paper is to comment on the above example of a wave equation and point out some serious problems associated with it.

If we require that the wave equation given by Khalil is derivable from an invariant Lagrangian it is necessary that a Hermitianising matrix A exists, i.e. a matrix satisfying the relations

$$A\Gamma_0 = \Gamma_0^\dagger A \qquad A^2 = 1 \qquad A = A^\dagger. \tag{5}$$

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The matrix A according to Gel'fand *et al* (1963) exists if in the representation (2) each irreducible component appears together with its conjugate. Thus $\dot{\tau}\sim(-1,2)$ and $\tau_2\sim(1,2)$ are conjugates of each other and τ_1, $\tau_1'\sim(0,2)$ are self-conjugate. The irreducible component $\tau_3\sim(1,3)$ appears in the representation without its conjugate component which is $\dot{\tau}_3\sim(-1,3)$ and is not self-conjugate either. As a result of this one cannot find a Hermitianising matrix for the given wave equation. If instead of the representation (2) one considers the extended representation $\dot{\tau}_2\oplus\tau_1\oplus\tau_2\oplus\tau_1\oplus\tau_3\oplus\dot{\tau}_3$ then a Hermitianising matrix can be found which, of course, will not refer to the above wave equation but to another. An implication of the fact that a Hermitianising matrix cannot be determined for the wave equation given by Khalil is that the charge

$$\int_U \langle\psi|A\Gamma_0|\psi\rangle\,\mathrm{d}U \tag{6}$$

associated with it cannot be determined either. Hence this wave equation is not suitable for the description of interactions such as those which are electromagnetic.

References

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